Energy Dynamics in a Simulation of LAPD Turbulence

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• A two fluid plasma model is used to simulate zero mean flow density gradient driven drift wave turbulence in the LArge Plasma Device (LAPD). The code used is BOUT++.

• Spectral energy dynamics are used to show where energy is injected and dissipated in the simulation, revealing a picture very different from what one would expect based on linear drift wave properties.

• We find that although a linear drift wave instability exists in the system, a nonlinear instability provides the dominant turbulent drive mechanism in the standard simulation. The nonlinear instability relies upon axial wavenumber transfer between finite and infinite wavelength modes.
LAPD is Suitable for Collisional Plasma Fluid Model

Machine and plasma size:
Plasma column length $\sim 17 \, m$
Plasma radius $\sim 30 \, cm$

Typical LAPD operational parameters:
$0.4 < B_0 < 2 \, kG$
$10^{11} < n_e < 4 \times 10^{12} \, cm^{-3}$
$0.5 < T_e < 8 \, eV$
$0.5 < T_i < 1.5 \, eV$
$f_{ci} \sim 400 \, KH\bar{z}$
$\nu_{in} \sim 2 \, KH\bar{z}$
$\nu_{ei} \sim 5 \, MH\bar{z}$
$\frac{\omega}{k_\parallel} \leq \nu_{the}$
$\nu_i/\omega_{ci} \sim 1$

$\lambda_{ei}/L_\parallel \sim 0.01$
$k_\perp \rho_i \sim 0.1$
Model Equations as Written for BOUT++

Continuity equation
\[
\frac{\partial N}{\partial t} = -v_E \cdot \nabla N_0 - N_0 \nabla \Vert v \Vert_e + \mu N \nabla^2 \perp N + S_N + \{\phi, N\}
\]

Parallel electron momentum equation
\[
\frac{\partial v_{\| e}}{\partial t} = -\frac{m_i}{m_e} \frac{T_{e0}}{N_0} \nabla \Vert N \Vert + \frac{m_i}{m_e} \nabla \Vert \phi \Vert - \nu_e v_{\| e} + \{\phi, v_{\| e}\}
\]

Energy balance equation
\[
\frac{\partial T_e}{\partial t} = -v_E \cdot \nabla T_{e0} - 1.71 \frac{2}{3} T_{e0} \nabla \Vert v \Vert_e + \frac{2}{3N_0} \kappa \Vert e \Vert T_e^2 - \frac{2m_e}{m_i} \nu_e T_e + \mu_T \nabla^2 \perp T_e + S_T + \{\phi, T_e\}
\]

Charge conservation / Vorticity equation
\[
\frac{\partial \varpi}{\partial t} = -N_0 \nabla \Vert v \Vert_e - \nu_{in} \varpi + \mu \phi \nabla^2 \perp \varpi + \{\phi, \varpi\}
\]
\[
\varpi = \nabla \perp \cdot (N_0 \nabla \perp \phi)
\]

- Electrostatic
- Only advective nonlinearities
- Artificial diffusions and viscosity.

Verification and validation studies:
- Popovich et al 2010, Umansky et al 2011
- Grid convergence study
- Friedman et al 2012
Experimental Profiles Used in Simulation

- Density equilibrium profile fit to experiment.
- Density source – subtracts m=0 density fluctuation component.

- $T_e$ is a typical looking tanh fit
- $T_i = 0$ eV

- Zero mean potential profile.
- Zonal flows evolved.

- Periodic axial BC. Dirichlet radial BC.
BOUT++ and LAPD Experimental Density Fluctuations Have Similar Statistical Properties

N Power Spectral Density

PDF

RMS N Fluctuation
Energy Clusters in n=0 Flute Structures Due to a Nonlinear Instability that Overcomes the Linear Drift Wave Instability

- Linear drift waves only inject energy at finite n
- n=0 flute modes are not a result of secondary instability, interchange instability, or KH instability. They are driven by a primary nonlinear instability
Total Nonconservative Energy Dynamics Show that Linear Stability Properties are Dominated by Self-Sustained Turbulent Dynamics

\[ \gamma(k) = \frac{\partial E_{tot}(k)/\partial t}{2E_{tot}(k)} \]
Main Energy Dynamic Mechanism Involves an Interplay of Density and Potential Fluctuations with Both \( n=0 \) and \( n \neq 0 \). Temperature Fluctuations Are Unimportant.

Biskamp et. al. 1995
The Removal of $n=0$ Modes Causes the Linear Instability to Dominate the Dynamics

$n=0$ removed at each timestep

$n=0$ removed, but zonal flow retained

Nonlinear phase
Linear phase
Without ZF
With ZF
The Removal of \( n=0 \) Modes Causes an Experimentally Inconsistent Peak in the Frequency Spectrum
Nonlinear Instabilities Can Overwhelm Linear Instabilities, Affecting Turbulent Characteristics

$$\alpha_{p\phi}(l) = \text{Im} \ln(\tilde{p}_{\phi}^* \tilde{\phi}_l)$$

Cross phase - measure of the instability drive and transport.

Pure drift wave turbulence

Pure ballooning mode turbulence

Drift-ballooning physics starting from small perturbation

B.D. Scott (2005)
Conclusion

• A nonlinear instability provides the dominant turbulent drive mechanism. The instability preferentially drives n=0 structures, but relies upon the nonlinearities and n=1 structures to access the adiabatic response.

• Removal of the n=0 modes causes the linear instability to dominate, but the turbulent frequency spectrum is more coherent, which is inconsistent with experiment.

• Nonlinear instability can be relevant in tokamak edge turbulence and linear growth rate calculations can be misleading when nonlinear instabilities are present.