2012 / 04 / 12

Nonlinear instabilities driven by coherent phase-space structures.

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NFRI

WCI Center for Fusion Theory

Background

- **Nonlinear wave-particle interactions** relevant to turbulence & transport
  - Energetic particle-driven Alfvén eigenmodes
  - Current-driven ion-acoustic instabilities which drive anomalous resistivity
  - Collisionless trapped electron mode
  - Trapped-ion ITG instabilities
  - etc.

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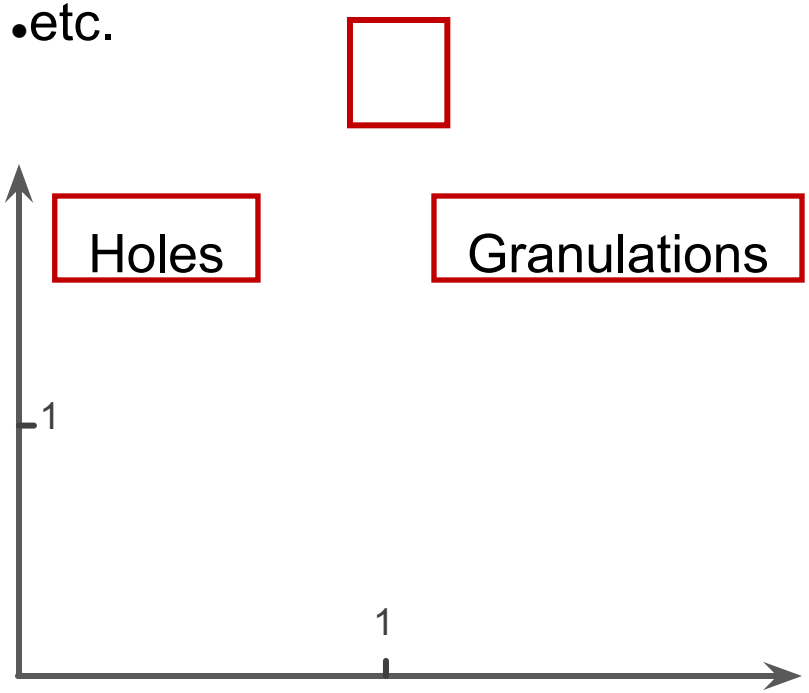


Motivation

■ **Phase-space structures** have deep implications for instabilities, turbulence and transport

- Drive nonlinear instabilities
- Intermittency in drift-wave turbulence
- Qualitative effect on nonlinear saturation
- Transport that departs from QL
- Drive zonal flow

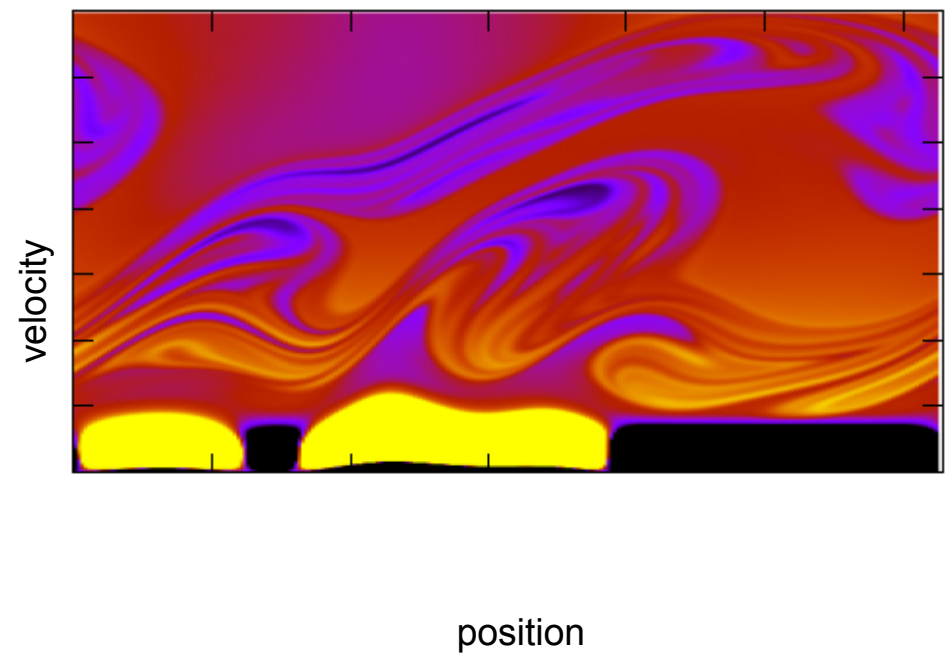
Dupree, et al., PF (72) *Berman, et al., PF (82)*
Terry, et al., PFB (90)
Berk, Breizman, et al., PRL (96)
Biglari, et al., PF (88) *Kosuga, et al., PoP (11)*
Kosuga, et al., in preparation

- *2:00 pm in Core Transport session (Ballroom B)*

■ Time is ripe to develop phase-space turbulence theory

Multiple-resonances B-o-T simulation:
1000 (space) x 1000 (velocity)
~ one day on 32 CPUs

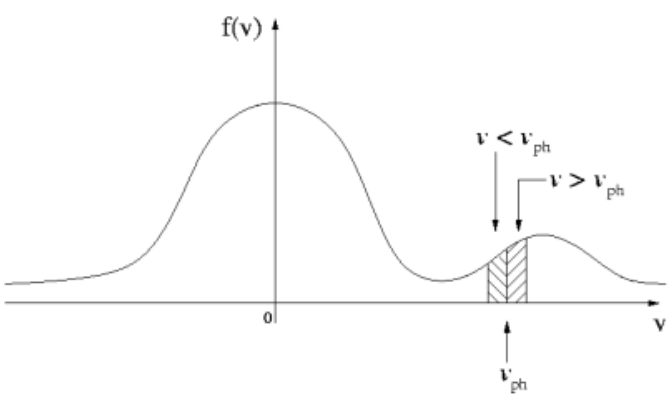
CDIA simulation:
32000 (space) x 2000 (velocity)
~ few days on 256 CPUs



Objectives

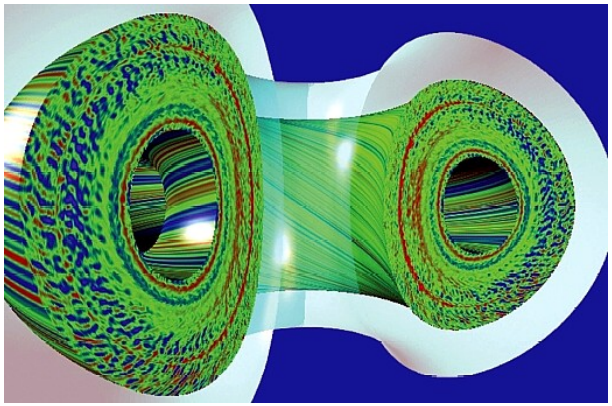
- **Understanding of nonlinear structure dynamics**
  - conditions for subcritical instabilities ?
  - mechanism of nonlinear structure growth ?
  - initial perturbation ?
  - nonlinear evolution of subcritical VS supercritical ?
  - how universal are nonlinear instabilities ?

- **Develop phase-space turbulence theory**
  - is there a clever way ?



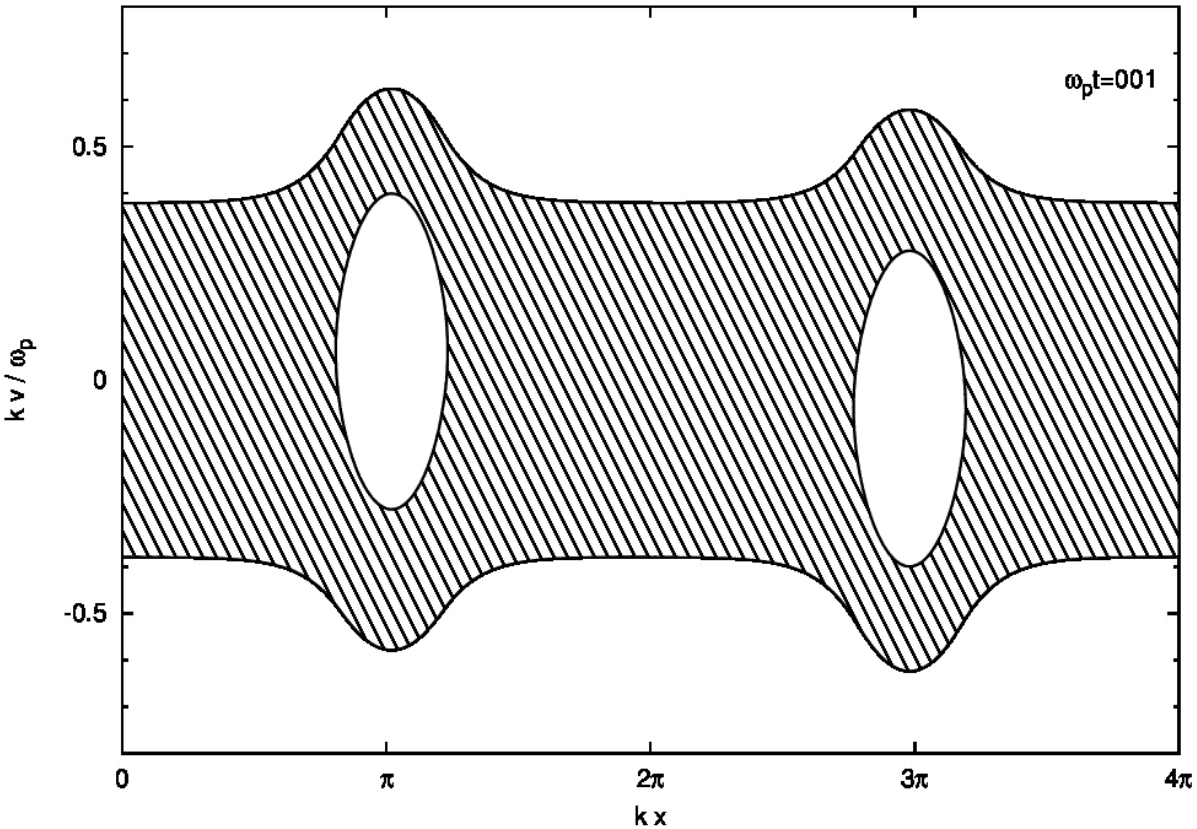
tractability

completeness



Approach

■ **First step:** single resonance, which features isolated, long-lived coherent structures (Kubo >> 1)



Self-binding structures (water-bag model with 2 BGK holes).

Roberts and Berk, PRL(67)
Berk, PF(70)

↔ 2D fluid in a gravitational field

↔ negative-mass instability

↔

■ **Bump-on-tail** (Berk-Breizman model) as a tractable model with key ingredients, and qualitative&quantitative similarities with experiments

Berk, Breizman, et al., PFB(90), PRL(96), PoP(99)

Outline

I. Berk-Breizman model
II. Phasestrophy growth
III. Nonlinear instabilities
IV. Phase-space turbulence

Summary

Perspectives

The BB model

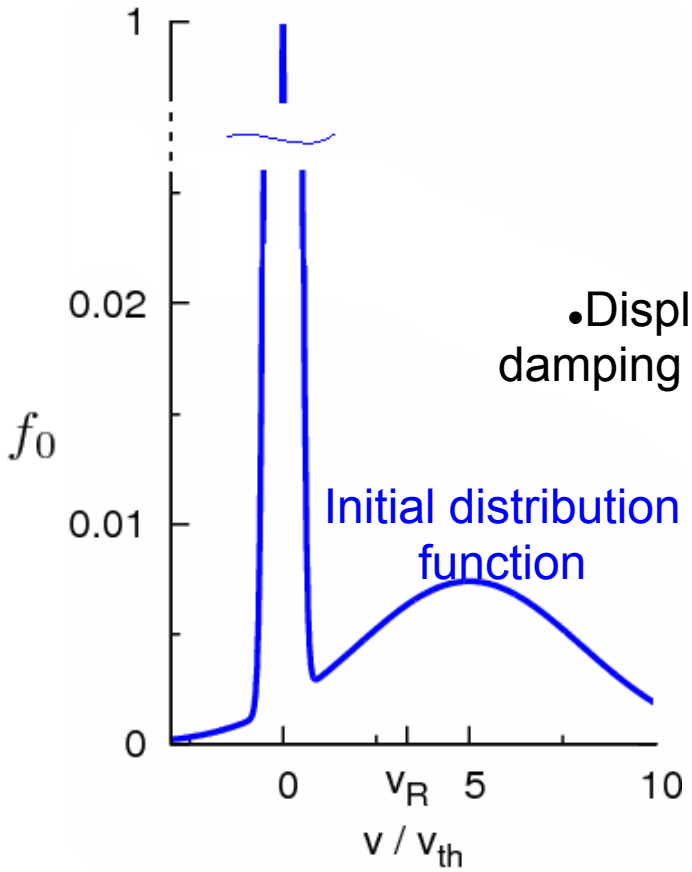
Classic “bump-on-tail” instability, with collisions and external damping.

Berk, et al., PoP(95)

- 1D kinetic equation with a collision operator including dynamical friction (drag), and velocity-space diffusion,

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + E\frac{\partial f}{\partial v} = C(f - f_0)$$

$$C(f - f_0) = \frac{\nu_f^2}{k}\frac{\partial\,(f - f_0)}{\partial v} + \frac{\nu_d^3}{k^2}\frac{\partial^2\,(f - f_0)}{\partial v^2}$$



- Displacement Current Equation with an external wave damping accounting for background dissipative mechanisms,

$$\frac{\partial E}{\partial t} = -4\pi q\int v(f - \bar{f})\mathrm{d}v \boxed{-2\gamma_d E}$$

- Single electrostatic wave,

$$E(x,t) = \hat{E}_k(t)e^{\imath kx} + \mathrm{c.c.}$$

Physical Interpretation

- Resonant particle trapping → island structure.
- Nonlinear evolution involves 6 time scales:

ω

ω_b $\qquad \omega_b \sim \sqrt{E}$

γ_L

≠ linear growth rate $\qquad \gamma \approx \gamma_L - \gamma_d$

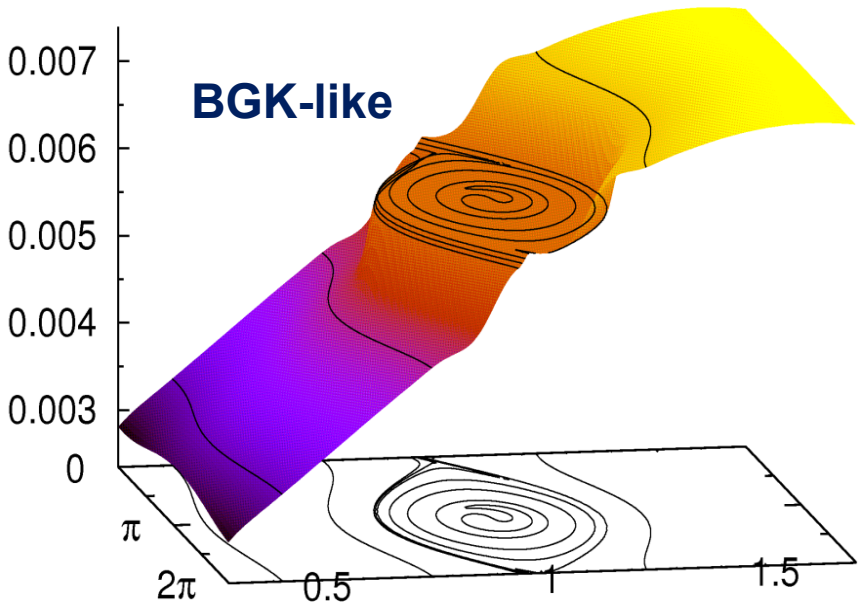
γ_d

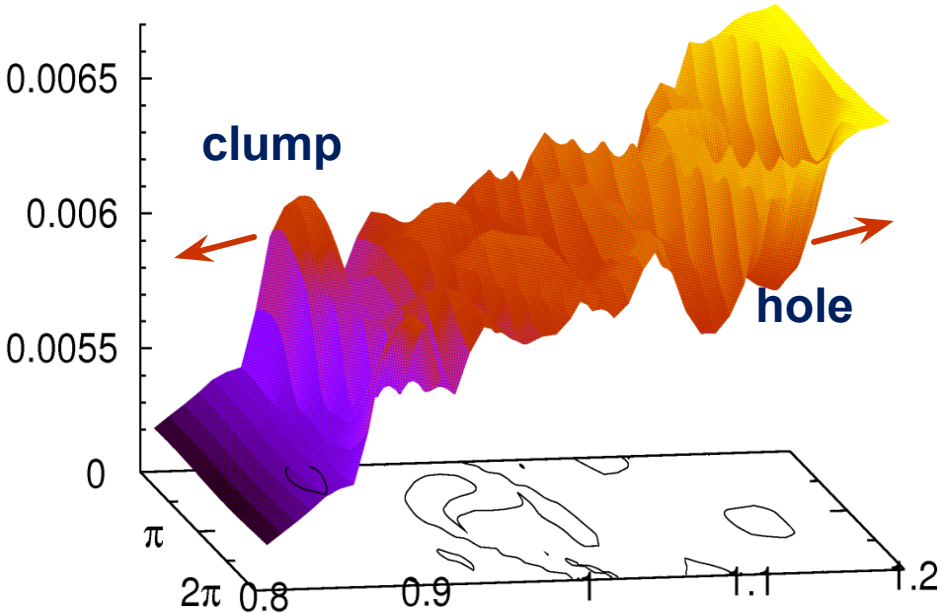
ν_f

ν_d

Holes and clumps

Berk, Breizman, Petviashvili, PLA(97)



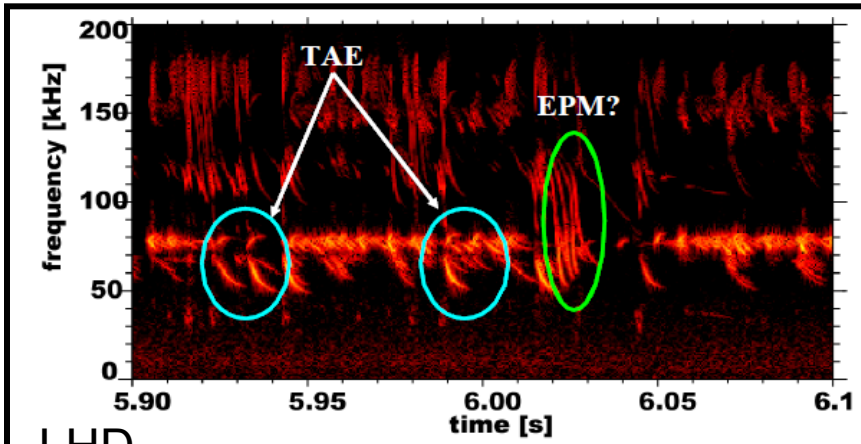


Chirping (frequency sweeping)

NL Chirping

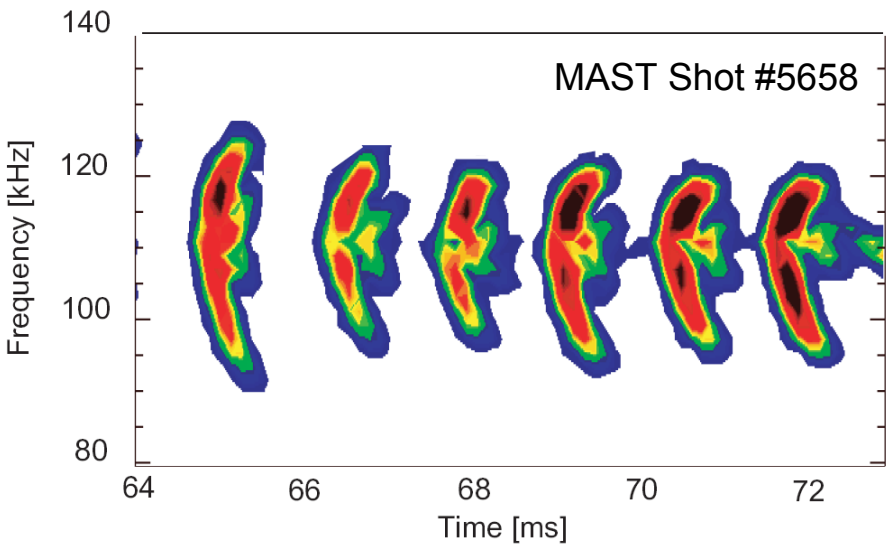
- Chaotic behavior,
- Splitting into two spectral modes, up and down in general,
- Lifetime << equilib. evolution time.
- Hole/clumps

Berk, et al., PLA(97)

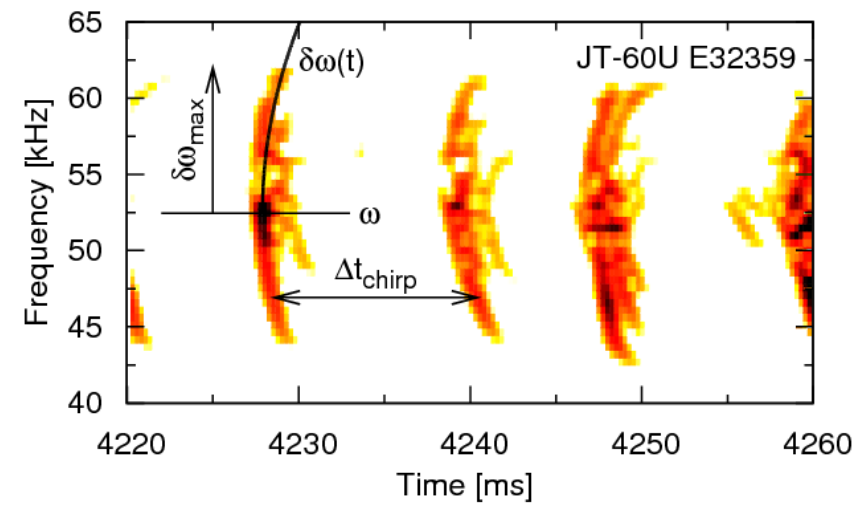


LHD

Toi, et al, PPCF(11).



Gryaznevich, Sharapov, PPCF(04)



Kusama, et al., NF(99)

Observations of chirping TAEs

Navigation

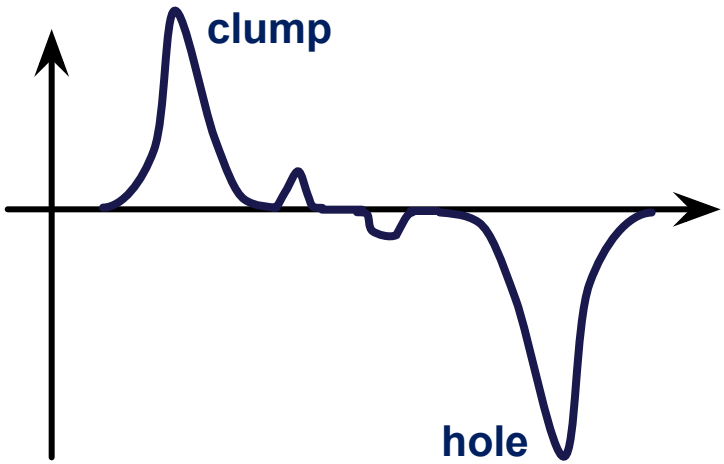
I. The Berk-Breizman model
II. Phasestrophy growth
III. Nonlinear instabilities
IV. Phase-space turbulence

Summary

Perspectives

Phasestrophy

$$f = f_0(v) + \delta f(x, v, t)$$



Phasestrophy is the phase-space density auto-correllation,

$$\Psi \equiv \int \langle \delta f^2 \rangle \, \mathrm{d}v$$

Diamond, Itoh[2], Modern Plasma Physics
Kosuga, Diamond, PoP 2011

In parallel with potential enstrophy in quasi-geostrophic systems,

$$\mathcal{E} \equiv \langle \delta q^2 \rangle$$

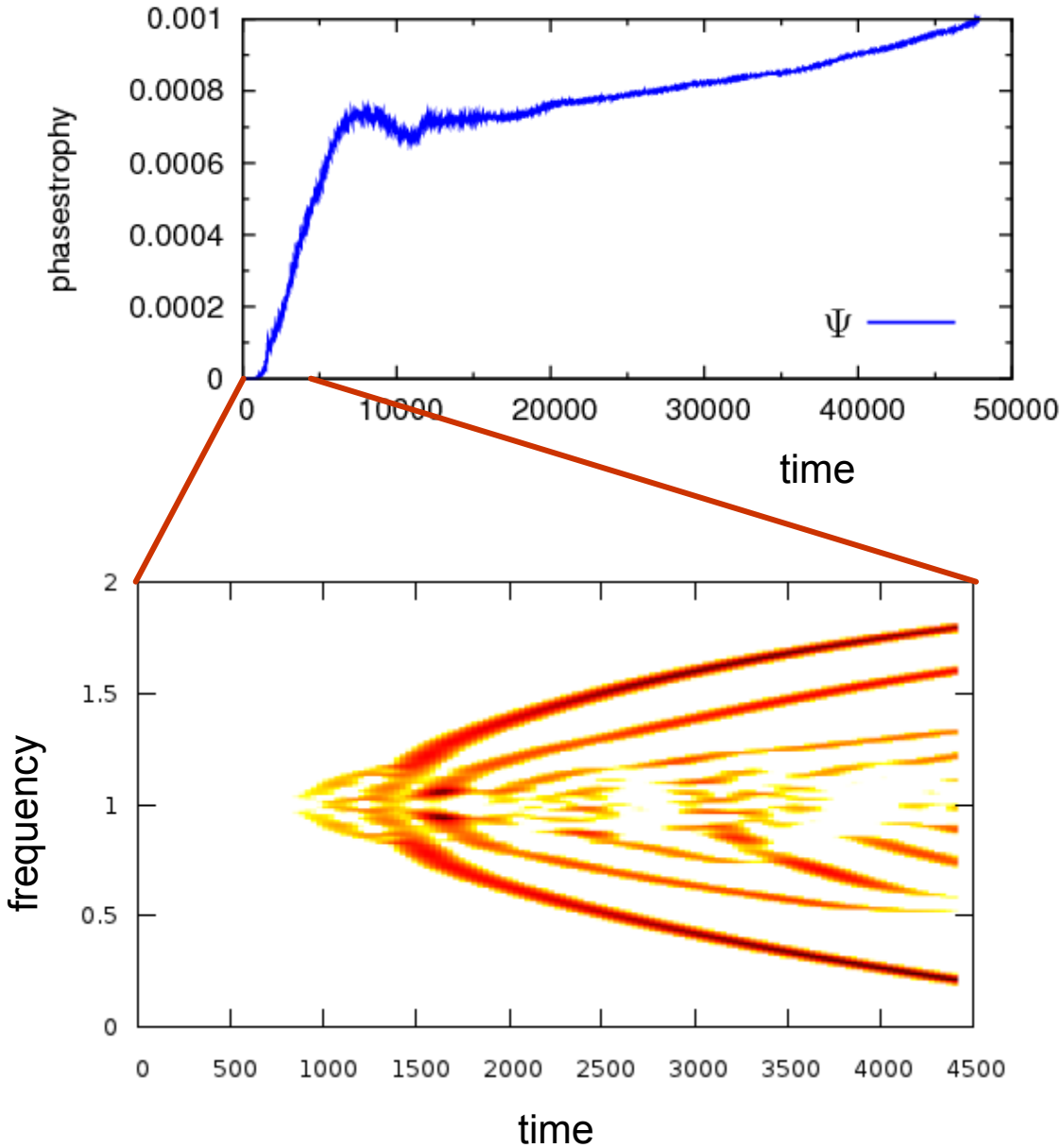
(q is the potential vorticity)

Relation with relative entropy:

If $f_0(v)$=const., $\Psi = 2 f_0 \, \delta S + O\left[\left(\frac{\delta f}{f_0}\right)^3\right]$

with $\delta S \equiv \int \left\langle f \log \frac{f}{f_0} \right\rangle \mathrm{d}v$

Phasestrophy growth



Total phasestrophy grows in time as holes and clumps are continuously created.

The growth is nonlinear.

Phasestrophy growth is proportional to effective force

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = -2\frac{\partial f_0}{\partial v}\int \langle E\delta f\rangle \,\mathrm{d}v$$

(assuming a constant slope for f_0)

Also proportional to change in momentum associated with h/c evolution

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = -2\frac{\partial f_0}{\partial v}\frac{\mathrm{d}\langle p\rangle}{\mathrm{d}t}$$

Hole/clump growth rate

Coupling phasestrophy equation

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = -2\frac{q}{m}\int_{-\infty}^{\infty}\frac{\mathrm{d}f_0}{\mathrm{d}v}\left\langle \tilde{E}\,\delta f\right\rangle \mathrm{d}v - \gamma_\Psi^{\mathrm{col}}\,\Psi,$$

$$\gamma_\Psi^{\mathrm{col}} = 2\nu_a + \frac{2}{\Psi}\frac{\nu_d^3}{k^2}\int_{-\infty}^{\infty}\left\langle\left(\frac{\partial\delta f}{\partial v}\right)^2\right\rangle \mathrm{d}v$$

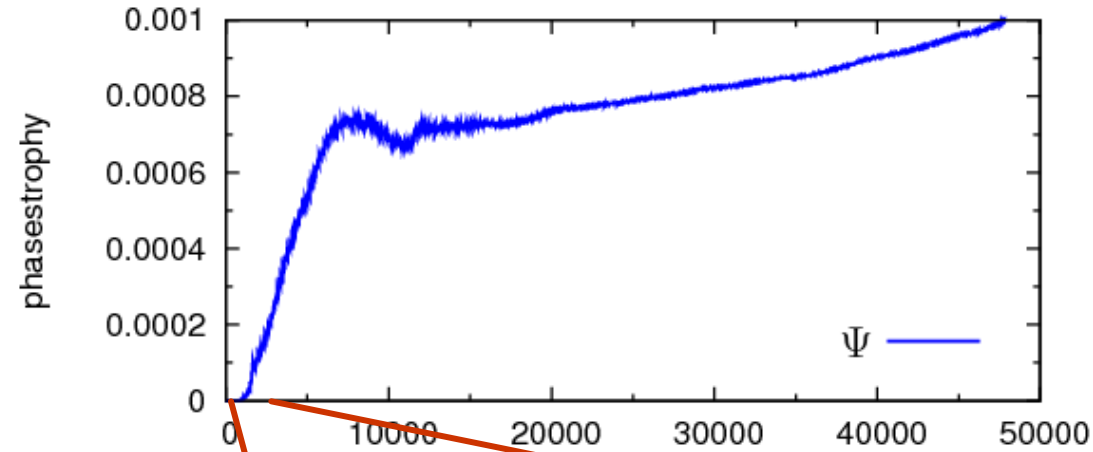
and wave-energy equation

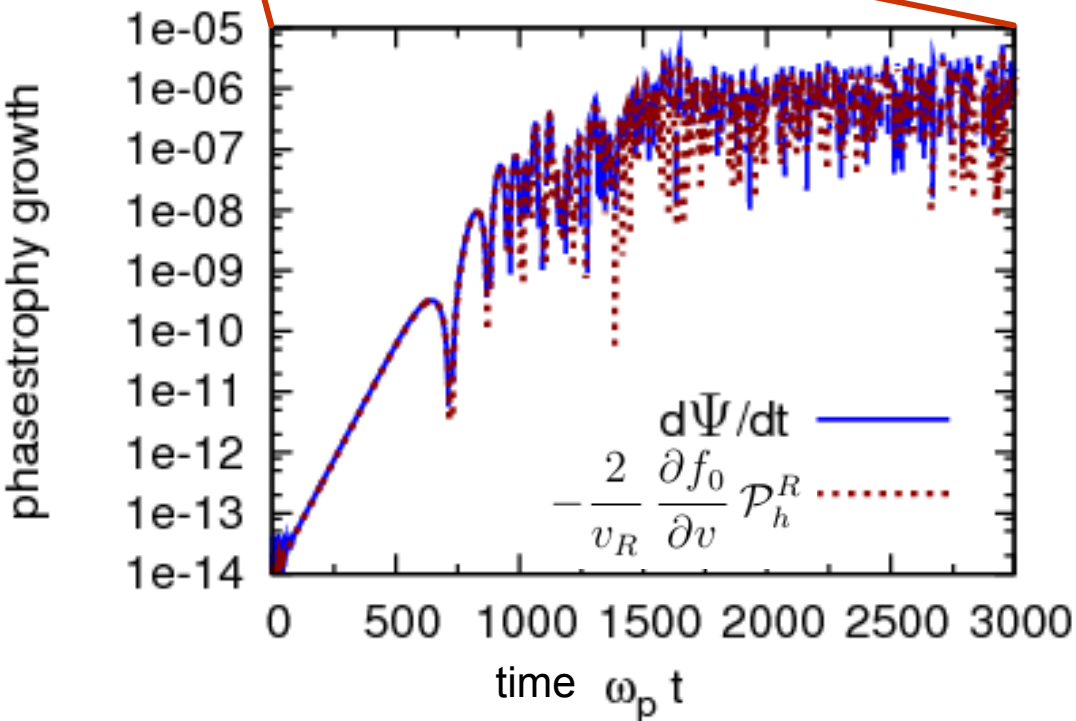
$$\frac{\mathrm{d}W}{\mathrm{d}t} + 2\gamma_d W = -\frac{q\omega_p}{k}\int\left\langle \tilde{E}\,\delta f\right\rangle \mathrm{d}v,$$

yields the kinetic counterpart of the
Charney-Drazin non-acceleration theorem

$$\left(\gamma_\Psi^{\mathrm{col}} + \frac{\mathrm{d}}{\mathrm{d}t}\right)\Psi = 2\frac{k}{m\omega_p}\frac{\mathrm{d}f_0}{\mathrm{d}v}\left(\frac{\mathrm{d}W}{\mathrm{d}t} + 2\gamma_d W\right).$$

Drive by dissipation





Resonant part of particle/wave energy transfer

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = -\frac{2}{v_R}\frac{\partial f_0}{\partial v}\mathcal{P}_h^R$$

$$= \frac{2}{v_R}\frac{\partial f_0}{\partial v}\left(\frac{1}{2}\frac{\mathrm{d}E_0^2}{\mathrm{d}t} + \gamma_d E_0^2\right)$$

$$= \frac{2}{v_R}\frac{\partial f_0}{\partial v}\left(\frac{\mathrm{d}\mathcal{W}}{\mathrm{d}t} + \mathcal{P}_d\right)$$

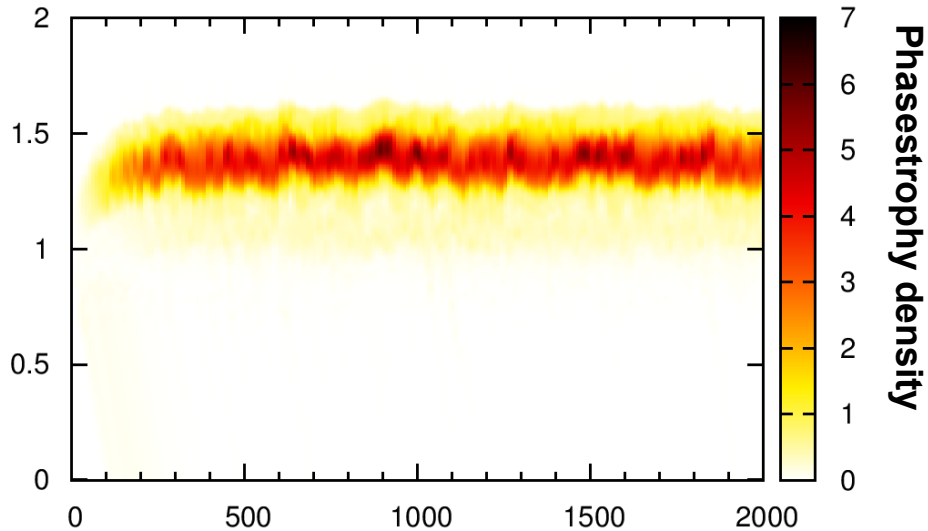
Variation of total wave energy, which includes sloshing energy.

Power due to dissipation

- $\frac{\mathrm{d}\Psi}{\mathrm{d}t} \sim \mathcal{P}_d$

in quasi steady-state

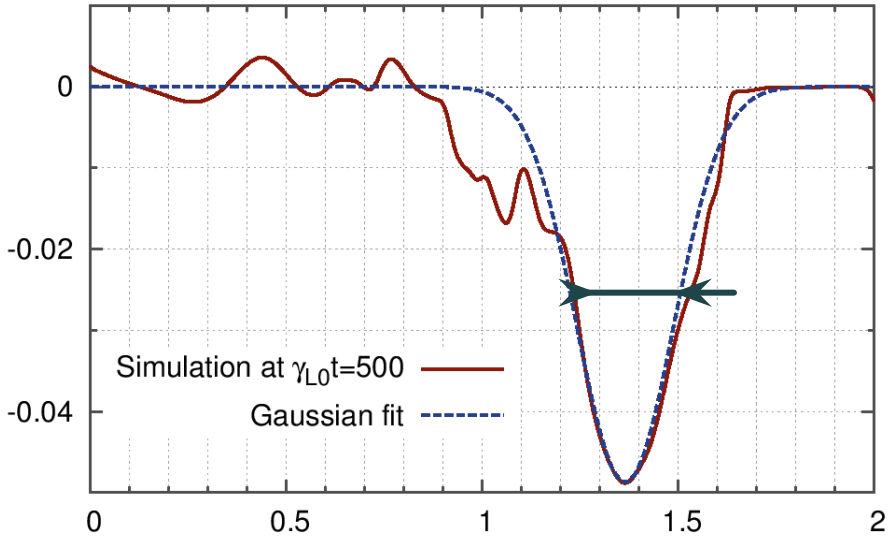
Single structure



Simulation designed such as a single hole dominates.

The hole can be modeled by a Gaussian.

- relation between W and Ψ (via Poisson)



Single structure growth rate

We obtain $$\frac{\mathrm{d}}{\mathrm{d}t}\Psi = \gamma_\Psi \Psi - \gamma_\Psi^{\mathrm{col}} \Psi,$$

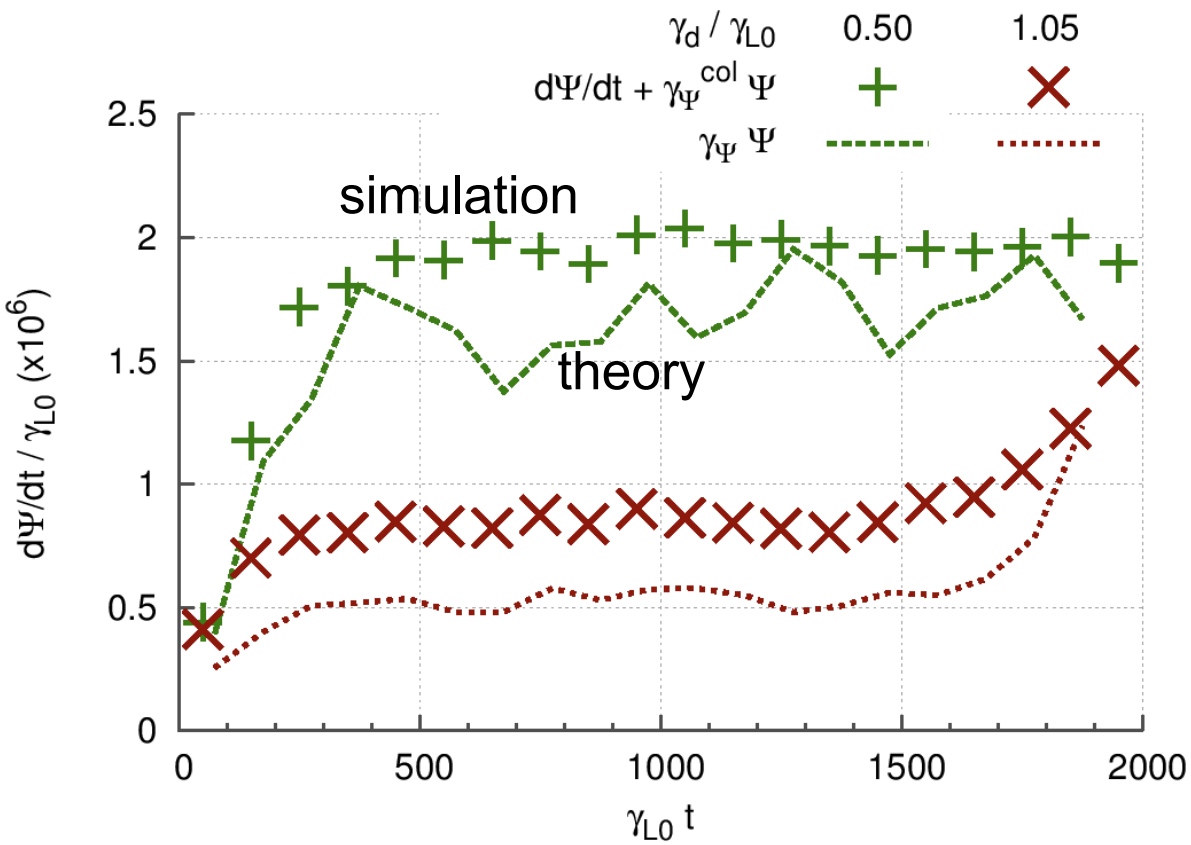
$$\boxed{\gamma_\Psi \approx \frac{16}{3\sqrt{\pi}} \frac{\Delta v}{v_R} \frac{\gamma_L}{\omega_p} \gamma_d.}$$

Nonlinear growth is independent of linear stability

- New understanding of subcritical instability

Nonlinear growth only requires

(and a large enough seed to counteract collisions)



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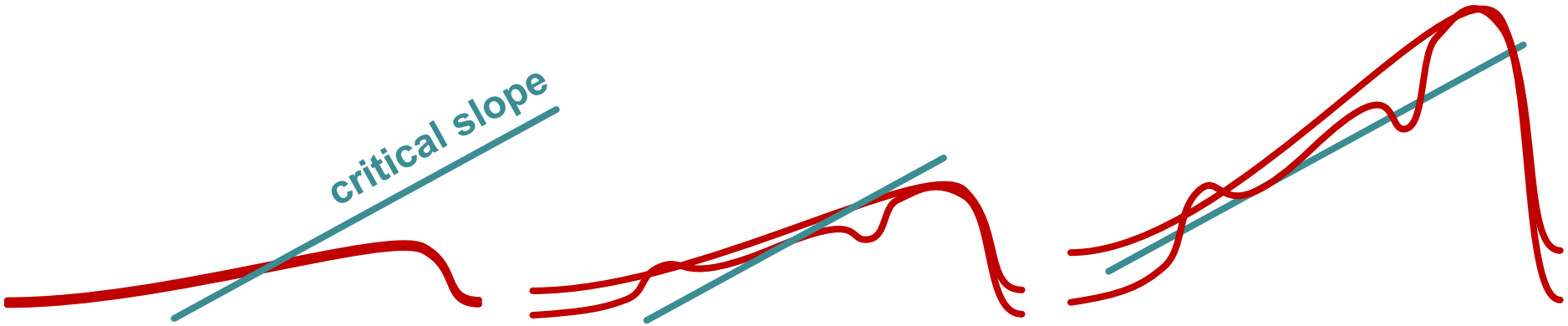
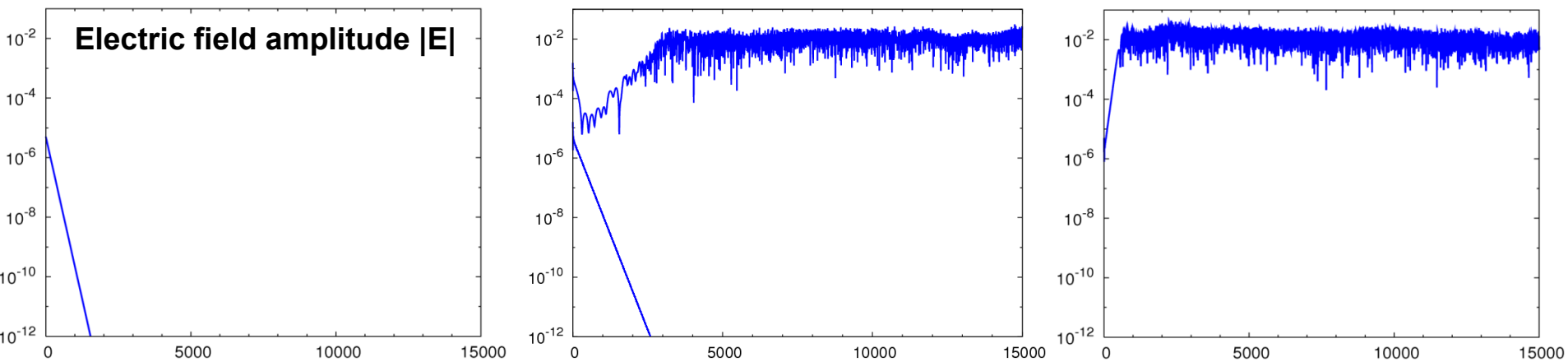
⚠ Subcritical instability



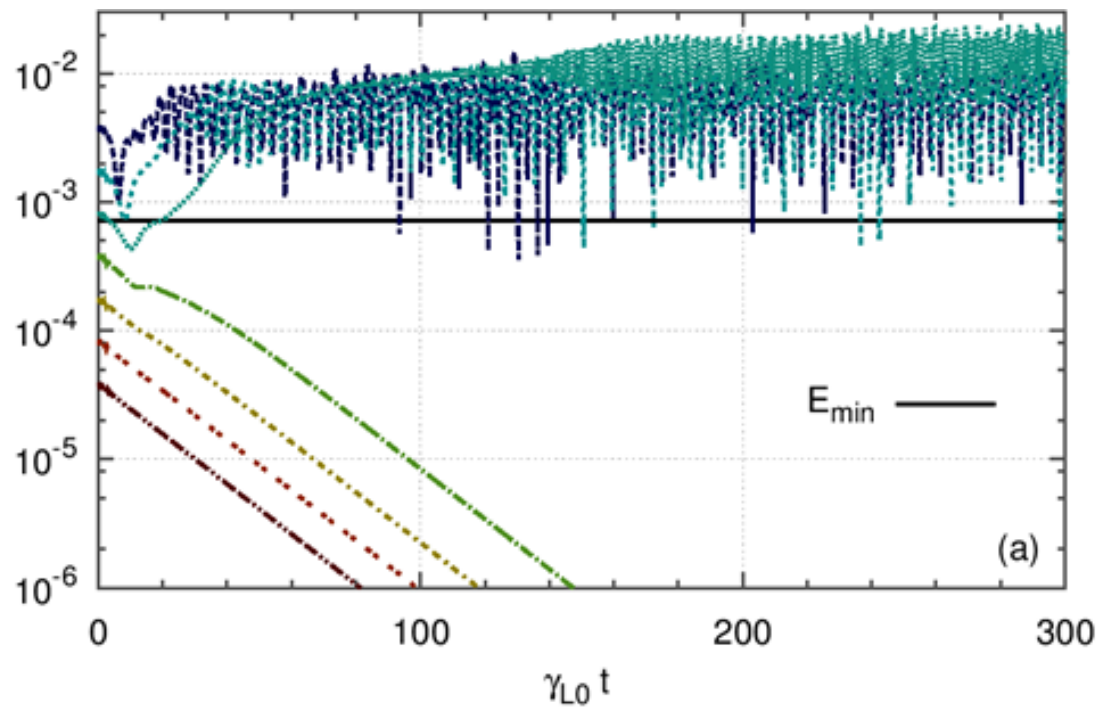
(Subcritical instability)



Seed structure

$$\frac{\mathrm{d}}{\mathrm{d}t}\Psi = \gamma_\Psi \Psi - \gamma_\Psi^{\mathrm{col}} \Psi,$$

$$\gamma_\Psi^{\mathrm{col}} \sim \nu_d^3/(k\Delta v)^2$$



Balancing growth by momentum exchange and decay by collisional diffusion yields the nonlinear instability threshold

$$E_{\min} \sim \left(\frac{\omega_p}{\gamma_{L0}}\frac{\omega_p}{\gamma_d}\right)^{2/3}\left(\frac{\nu_d}{\omega_p}\right)^2$$

Barely unstable

$$\frac{\mathrm{d}}{\mathrm{d}t}\Psi = \gamma_\Psi \Psi - \gamma_\Psi^{\mathrm{col}} \Psi, \qquad \Psi \sim \Delta v^3$$

Nonlinearly,

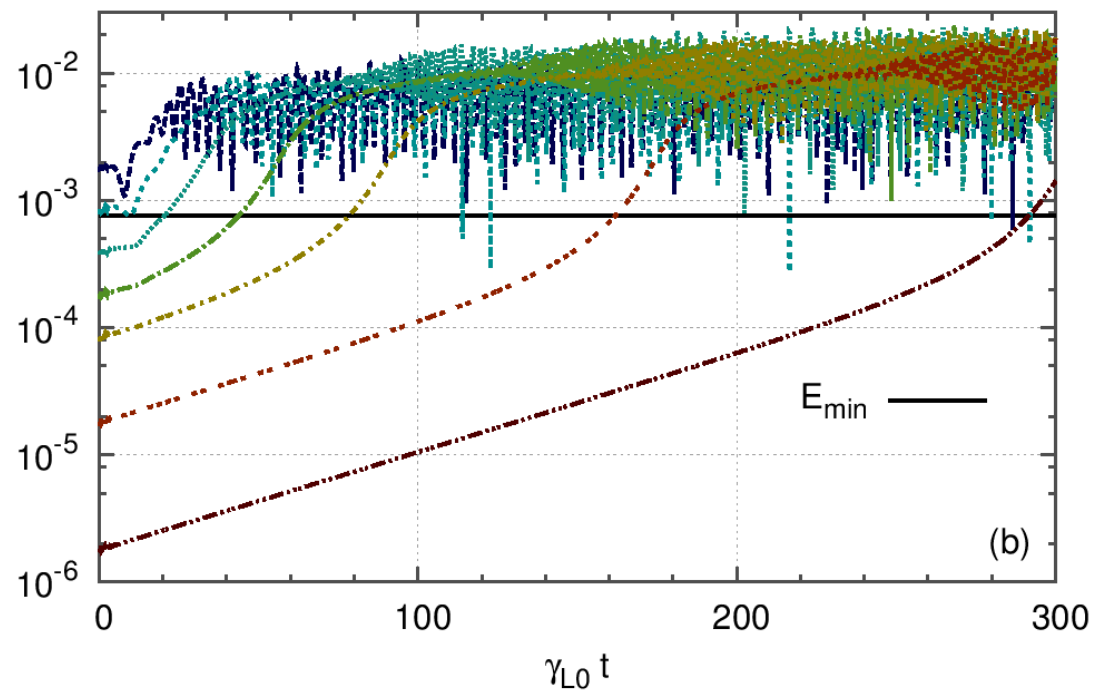
$$\mathring{\Delta} v / \Delta v = \frac{\gamma_\Psi - \gamma_\Psi^{\mathrm{col}}}{3}$$

Linearly,

$$\mathring{\Delta} v / \Delta v = \gamma/2$$

- Theory predicts the existence of a new “barely unstable” nonlinear instability when

$$\gamma_\Psi - \gamma_\Psi^{\mathrm{col}} > (3/2)\gamma$$



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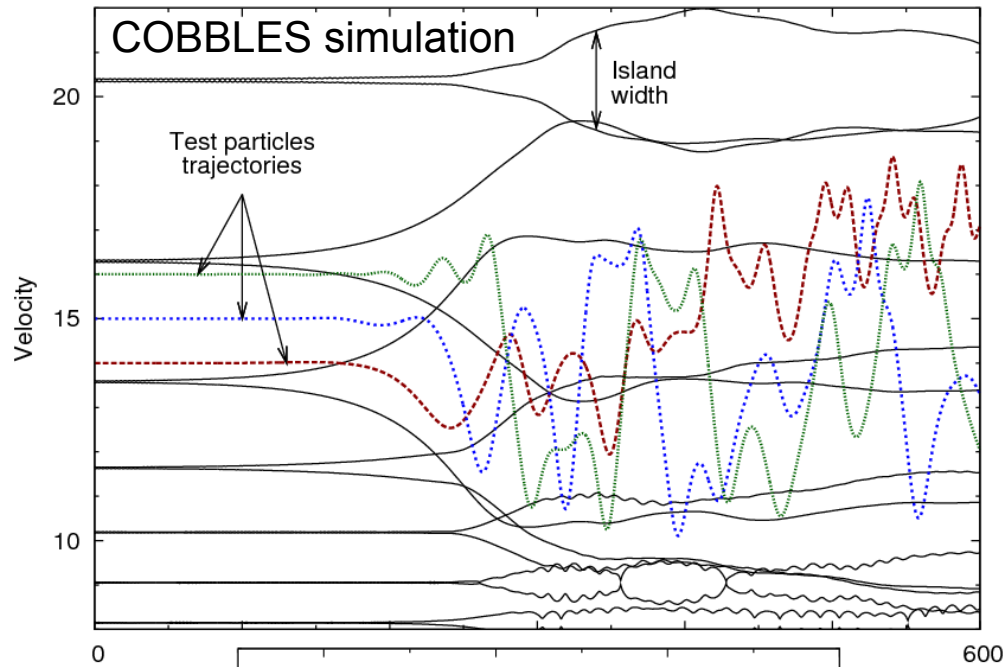
Collisionless, γ_d=0 case

Without extrinsic dissipation, phase-space structures are absent and quasilinear theory is valid.

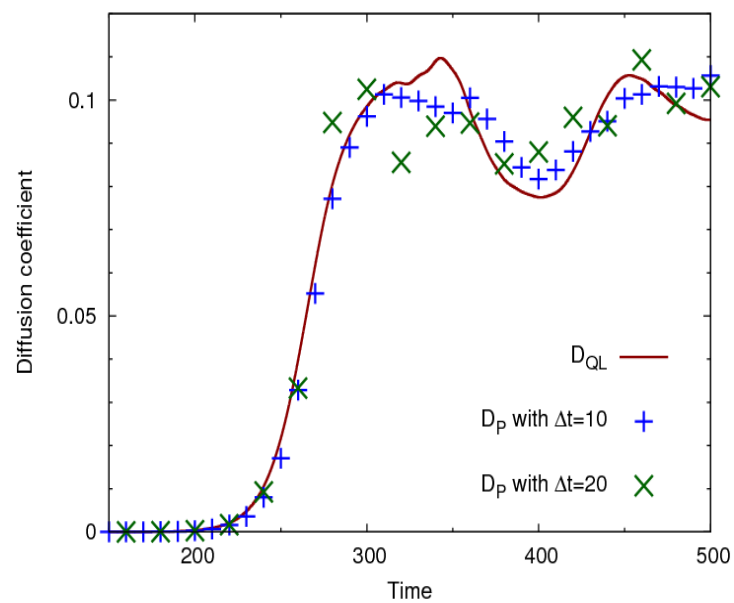


Diffusion of test particles (300'000 particles),

$$D_{\mathrm{P}} = \frac{\left\langle [v(t_0+\Delta t) - v(t_0)]^2 \right\rangle}{2\Delta t}$$

agrees with quasilinear theory,

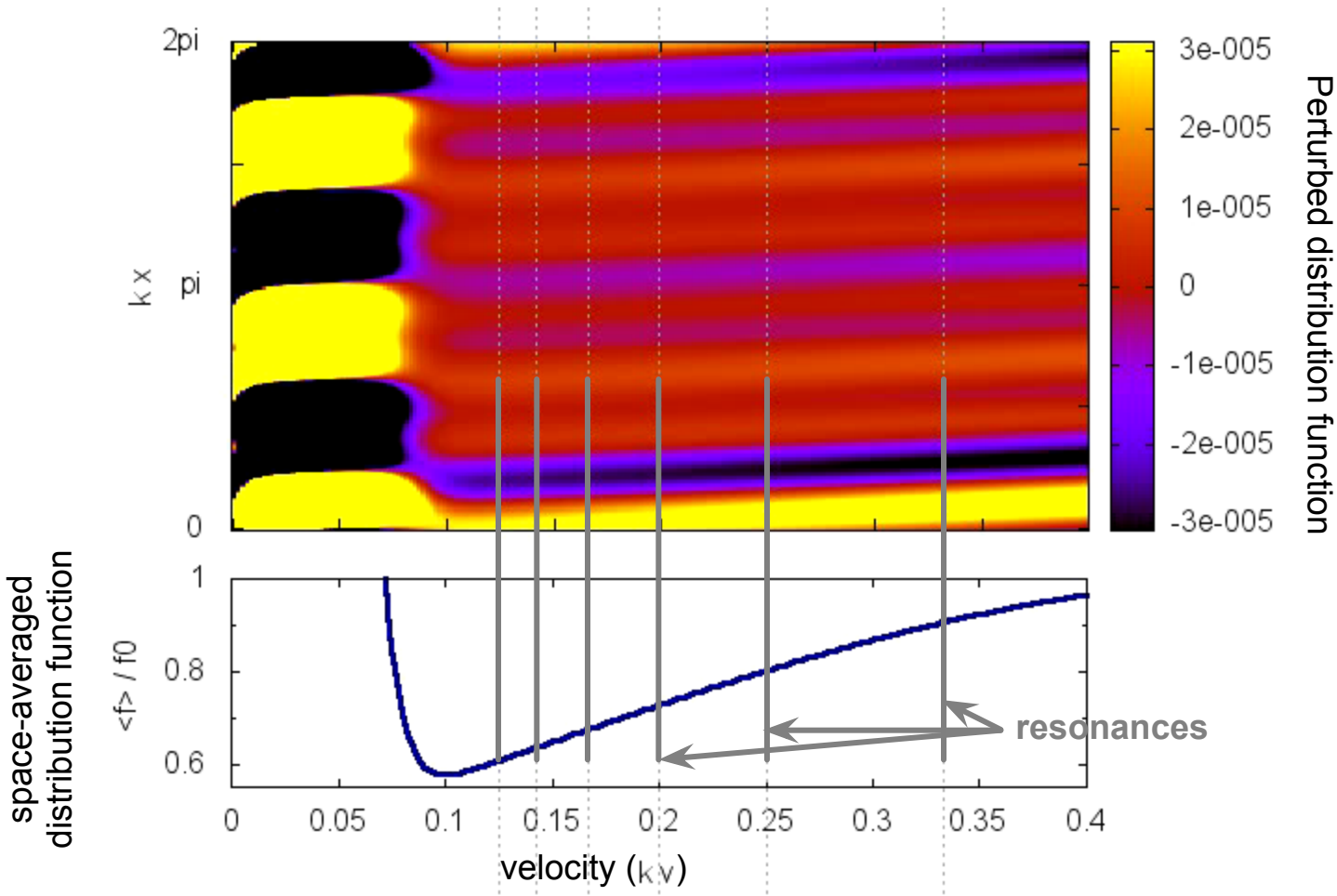
$$D_{\mathrm{QL}} = \frac{\pi \sum_m |E_{k_m}|^2 / k_m}{\Delta v_R}$$



Sagdeev & Galeev, Nonlinear Plasma Theory (69)

Phase-space turbulence

With extrinsic dissipation, existence of coherent phase-space structures, which disappear on a collisional diffusion time ~ 1000 >> quasilinear diffusion time ~ 100.



Extension of quasi-linear theory with interacting phase-space structures.

- **"phase-space turbulence"**

In a nutshell

- Growth of phase-space structures, quantified by phasestrophy growth, corresponds to momentum exchange driven by external damping.
- For a single hole or clump, a new theory gives a simple expression for the structure growth rate, $\gamma_{hc} \sim \gamma_L \gamma_d$. Quantitative agreement with simulations. Need closure theory for multiple holes/clumps.
- New interpretation for nonlinear drive of instability.
- First observation of nonlinear instability in the barely unstable regime.

Take-out message

Importance of phase-space structures: can drive instabilities, transport, ZF, …

Perspectives

- Similar work for the CDIA, effect on anomalous resistivity.
- Application to drift wave holes and their interaction with zonal flow ->
- Phase-space turbulence theory.

IRFM cea cadarache